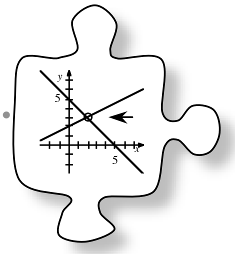
6.2.3 Can I solve without substituting?



Solving Systems Using Elimination

In this chapter, you have learned the Substitution Method for solving systems of equations. In Chapter 4, you learned the Equal Values Method. But are these methods the best to use for all types of systems? Today you will develop a new solution method that can save time for systems of equations in standard form.

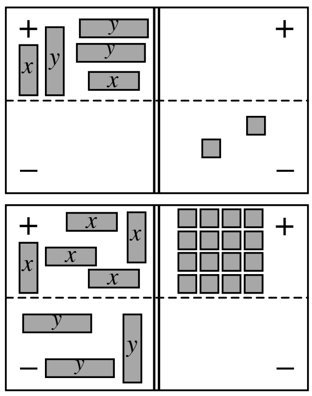
6-56. Jeanette is trying to find the intersection point of these two equations:

$$2x+3y=-2$$
$$5x-3y=16$$

She has decided to use substitution to find the point of intersection. Her plan is to solve the first equation for *y*, and then to substitute the result into the second equation. Use Jeanette's idea to solve the system.

6-57. AVOIDING THE MESS: THE ELIMINATION METHOD

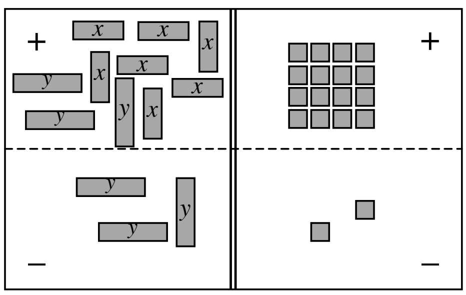
Your class will now discuss a new method, called the **Elimination Method**, to find the solution to Jeanette's problem without the complications and fractions of the previous problem. Your class discussion is outlined below.



a. Verify that each equation mat at right represents one of Jeanette's equations.

$$2x+3y=-2$$
$$5x-3y=16$$

b. Can these two equations be merged onto one equation mat as shown below? That is, can the left sides and right sides of two equations be added together to create a new equation? Why or why not?



This is the result when the equations are combined.

6-57. *Problem continued from previous page.*

c. Write a new equation for the result of merging Jeanette's equations. Simplify and then solve this new equation for the remaining variable. Notice that you now have only one equation with one variable. What happened to the y-terms?

d. Use your solution for x to find y. Check to be sure your solution makes both original equations true.

e. How can you record this process on paper? That is, when solving this type of system, how can you show that you are combining the equations?

f. Now use the Elimination Method to solve the system of equations at right for x and y. Check your solution.

$$\begin{aligned} 2x - y &= -2 \\ -2x + 3y &= 10 \end{aligned}$$

6-58. Pat was in a fishing competition at Lake Pisces. She caught some bass and some trout. Each bass weighed 3 pounds, and each trout weighed 1 pound. Pat caught a total of 30 pounds of fish. She got 5 points in the competition for each bass, but since trout are endangered in Lake Pisces, she lost 1 point for each trout. Pat scored a total of 42 points.

a. Write a system of equations representing the information in this problem.

b. Is this system a good candidate for the Elimination Method? Why or why not?

c. Solve this system to find out how many bass and trout Pat caught. Be sure to record your work and check your answer by substituting your solution into the original equations.

6-59. ANNIE NEEDS YOUR HELP

Annie was all ready to "push together" the two equations below to eliminate the x-terms when she noticed a problem: Both x-terms are positive!

$$\begin{aligned} 2x + 7y &= 13 \\ 2x + 3y &= 5 \end{aligned}$$

With your team, figure out something you could do that would allow you to put these equations together and eliminate the x-terms. As you try out different ideas, ask your teacher for some algebra tiles and an equation mat if you think they will help. Once you have figured out a method, solve the system and check your solution. Be ready to share your method with the class.

6-60. Find the point of intersection of each pair of lines below. If you use an equation mat, be sure to record your process on paper. Otherwise, show your steps algebraically. Check each solution when you are finished.

a. $2y - x = 5$
$-3y + x = -9$

b. $2x - 4y = 14$
$4y - x = -3$

c. $3x + 4y = 1$
$2x + 4y = 2$

MATH NOTES

METHODS AND MEANINGS

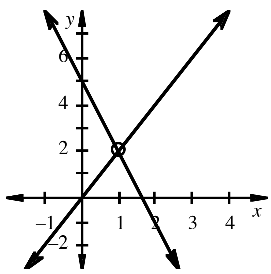
Systems of Linear Equations

A **system of linear equations** is a set of two or more linear equations that are given together, such as the example at right:

$$y = 2x$$
$$y = -3x + 5$$

If the equations come from a real-world context, then each variable will represent some type of quantity in both equations. For example, in the system of equations above, y could stand for a number of dollars in *both* equations.

To represent a system of equations graphically, you can simply graph each equation on the same set of axes. The graph may or may not have a **point of intersection**, as shown circled at right.



Sometimes two lines have *no* points of intersection. This happens when the two lines are parallel. It is also possible for two lines to have an *infinite* number of intersections. This happens if they are simply the same equation in different forms. Such lines are said to **coincide**.

Also notice that the point of intersection lies on *both* graphs in the system of equations. This means that the point of intersection is a **solution** to *both* equations in the system. For example, the point of intersection of the two lines graphed above is (1, 2). This point of intersection makes both equations true, as shown at right.

$$y = 2x \qquad y = -3x + 5$$
$$(2) = 2(1) \qquad (2) = -3(1) + 5$$
$$2 = 2 \qquad 2 = -3 + 5$$
$$2 = 2$$

The point of intersection makes both equations true; therefore the point of intersection is a solution to both equations. For this reason, the point of intersection is sometimes called a **solution to the system of equations**.

Review & Preview

6-61. Find the point of intersection of each pair of lines, if one exists. If you use an equation mat, be sure to record your process on paper. Check each solution, if possible.

a. $x = -2y - 3$
$4y - x = 9$

b. $x + 5y = 8$
$-x + 2y = -1$

c. $4x - 2y = 5$
$y = 2x + 10$

6-62. Jai was solving the system of equations below when something strange happened.

$$y = -2x + 5$$
$$2y + 4x = 10$$

a. Solve the system. Explain to Jai what the solution should be.

b. Graph the two lines on the same set of axes. What happened?

c. Explain how the graph helps to explain your answer in part (a).

6-63. On Tuesday the cafeteria sold pizza slices and burritos. The number of pizza slices sold was 20 less than twice the number of burritos sold. Pizza sold for \$2.50 a slice and burritos for \$3.00 each. The cafeteria collected a total of \$358 for selling these two items.

a. Write two equations with two variables to represent the information in this problem. Be sure to define your variables.

b. Solve the system from part (a). Then determine how many pizza slices were sold.

6-64. A local deli sells 6-inch sub sandwiches for \$2.95. It has decided to sell a "family sub" that is 50 inches long. How much should it charge? Show all work.

6-65. Represent the tile pattern below with a table, a rule, and a graph.

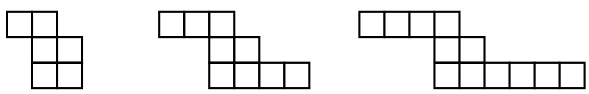


Figure 1 Figure 2 Figure 3

6-66. Use generic rectangles to multiply each of the following expressions.

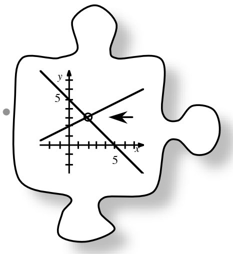
a. $(x+2)(x-5)$

b. $(y+2x)(y+3x)$

c. $(3y-8)(-x+y)$

d. $(x-3y)(x+3y)$

6.2.4 How can I eliminate a variable?



More Elimination

In Lesson 6.2.3, you learned how to use the Elimination Method to solve systems of equations. In this method, you combined two equations in a way that made one variable disappear. This method is particularly useful for solving systems of equations where neither equation is in $y = mx + b$ form.

Today you will practice using the Elimination Method while learning to deal with various complications that systems of equations sometimes present. As you solve these systems, ask your teammates these questions:

How can you create one equation with only one variable?

How can you eliminate one variable?

How do you know your solution is correct?

6-67. Which system of equations below would be easiest to solve using the Elimination Method? Once you have explained your decision, use the Elimination Method to solve this system of equations. (You do not need to solve the other system!) Record your steps and check your solution.

a. $\begin{aligned} 5x - 4y &= 37 \\ -8x + 4y &= -52 \end{aligned}$

b. $\begin{aligned} 4 - 2x &= y \\ 3y + x &= 11 \end{aligned}$

6-68. Rachel is trying to solve this system:

$$\begin{aligned} 2x + y &= 10 \\ 3x - 2y &= 1 \end{aligned}$$

a. Combine these equations. What happened?

b. Is $2x + y = 10$ the same line as $4x + 2y = 20$? That is, do they have the same solutions? Are their graphs the same? **Justify** your conclusion! Be ready to share your reasoning with the class.

c. Since you can rewrite $2x + y = 10$ as $4x + 2y = 20$, perhaps this equivalent form of the original equation can help solve this system. Combine $4x + 2y = 20$ and $3x - 2y = 1$. Is a variable eliminated? If so, solve the system for *x* and *y*. If not, brainstorm another way to eliminate a variable. Be sure to check your solution.

d. Why was the top equation changed? Would a variable have been eliminated if the bottom equation were multiplied by 2 on both sides? Test this idea.

6-69. For each system below, determine:

- Is this system a good candidate for the Elimination Method? Why or why not?
- What is the best way to get one equation with one variable? Carry out your plan and solve the system for both variables.
- Is your solution correct? Verify by substituting your solution into both original equations.

a. $\begin{aligned} 5m+2n&=-10 \\ 3m+2n&=-2 \end{aligned}$

b. $\begin{aligned} 6a-b&=3 \\ b+4a&=17 \end{aligned}$

c. $\begin{aligned} 7x+4y&=17 \\ 3x-2y&=-15 \end{aligned}$

d. $\begin{aligned} -18x+3y&=-12 \\ 6x-y&=4 \end{aligned}$

6-70. A NEW CHALLENGE

Carefully examine this system:

$$\begin{aligned} 4x+3y&=10 \\ 9x-4y&=1 \end{aligned}$$

With your team, propose a way to combine these equations so that you eventually have one equation with one variable. Be prepared to share your proposal with the class.

METHODS AND MEANINGS

MATH NOTES

Coefficients and Constants

A **coefficient** is the numerical part of a term that includes a variable. For example, in the expression below, the coefficient of $7x^2$ is the number 7, the coefficient of $4x$ is 4, and the coefficient of $-y$ is -1. Note that the 9 in the expression below is called a **constant**. A constant is a term that does not include a variable.

$$7x^2+4x-y+9$$

Review & Preview

6-71. Solve these systems of equations using any method. Check each solution, if possible.

a. $$\begin{aligned} 2x + 3y &= 9 \\ -3x + 3y &= -6 \end{aligned}$$

b. $$\begin{aligned} x &= 8 - 2y \\ y - x &= 4 \end{aligned}$$

c. $$\begin{aligned} y &= -\tfrac{1}{2}x + 7 \\ y &= x - 8 \end{aligned}$$

d. $$\begin{aligned} 9x + 10y &= 14 \\ 7x + 5y &= -3 \end{aligned}$$

6-72. For each line below, make a table and graph. What do you notice?

a. $y = \frac{2}{3}x - 1$

b. $2x - 3y = 3$

6-73. **Consecutive numbers** are integers that are in order without skipping, such as 3, 4, and 5. Find three consecutive numbers with a sum of 54.

6-74. Identify the hypothesis and conclusion for each of the following statements. Then decide if the statement is true or false. Justify your decision. You may want to review the meanings of hypothesis and conclusion from problem 6-31.

a. If $y = \frac{2}{3}x - 5$, then the point (6, –1) is a solution.

b. If Figure 2 of a tile pattern has 13 tiles and Figure 4 of the same pattern has 15 tiles, then the pattern grows by 2 tiles each figure.

c. If $(3x + 1)(x - 2) = 4$, then $3x^2 - 5x - 2 = 4$.

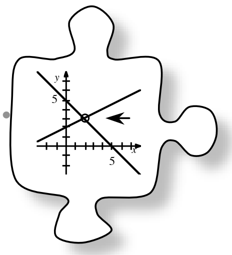
6-75. Aimee thinks the solution to the system below is (–4, –6). Eric thinks the solution is (8, 2). Who is correct? Explain your reasoning.

$$\begin{aligned} 2x - 3y &= 10 \\ 6y &= 4x - 20 \end{aligned}$$

6-76. Figure 3 of a tile pattern has 11 tiles, while Figure 4 has 13 tiles. If the tile pattern grows at a constant rate, how many tiles will Figure 50 have?

6.2.5 What is the best method?

Choosing a Strategy for Solving Systems



When you have a system of equations to solve, how do you know which method to use? Focus today on how to choose a strategy that is the most convenient, efficient, and accurate for a system of equations.

6-77. Erica works in a soda-bottling factory. As bottles roll past her on a conveyer belt, she puts caps on them. Unfortunately, Erica sometimes breaks a bottle before she can cap it. She gets paid 4 cents for each bottle she successfully caps, but her boss deducts 2 cents from her pay for each bottle she breaks.

Erica is having a bad morning. Fifteen bottles have come her way, but she has been breaking some and has only earned 6 cents so far today. How many bottles has Erica capped and how many has she broken?

a. Write a system of equations representing this situation.

b. Solve the system of equations using *two* different methods: substitution and elimination. Demonstrate that each method gives the same answer.

6-78. For each system below, decide which strategy to use. That is, which method would be the most efficient, convenient, and accurate: the Substitution Method, the Elimination Method, or the Equal Values Method? Do not solve the systems yet! Be prepared to **justify** your reasons for choosing one strategy over the others.

a. $x = 4 - 2y$
$3x - 2y = 4$

b. $3x + y = 1$
$4x + y = 2$

c. $x = -5y + 2$
$x = 3y - 2$

d. $2x - 4y = 10$
$x = 2y + 5$

e. $y = \frac{1}{2}x + 4$
$y = -2x + 9$

f. $-6x + 2y = 76$
$3x - y = -38$

g. $5x + 3y = -6$
$2x - 9y = 18$

h. $x - 3 = y$
$2(x - 3) - y = 7$

6-79. Your teacher will assign you a variety of systems from problem 6-78 to solve. With your team, use the best strategy to solve each system assigned by your teacher. Be sure to check your solution.

6-80. In your Learning Log, write down everything you know about solving systems of equations. Include examples and explain your reasoning. Title this entry "Solving Systems of Equations" and label it with today's date.

METHODS AND MEANINGS

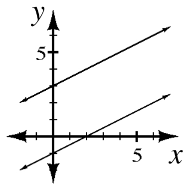
MATH NOTES

Intersection, Parallel, and Coincide

When two lines lie on the same flat surface (called a plane), they may **intersect** (cross each other) once, an infinite number of times, or never.

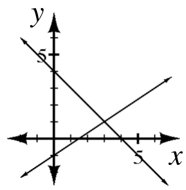
For example, if the two lines are **parallel**, then they never intersect. Examine the graph of two parallel lines at right. Notice that the distance between the two lines is constant.

parallel lines



However, what if the two lines lie exactly on top of each other? When this happens, we say that the two lines **coincide**. When you look at two lines that coincide, they appear to be one line. Since these two lines intersect each other at all points along the line, coinciding lines have an infinite number of intersections.

intersecting lines



While some systems contain lines that are parallel and others coincide, the most common case for a system of equations is when the two lines intersect once, as shown at right.

Review & Preview

6-81. Solve the following systems of equations using any method. Check each solution, if possible.

a. $$\begin{aligned} -2x + 3y &= 1 \\ 2x + 6y &= 2 \end{aligned}$$

b. $$\begin{aligned} y &= \tfrac{1}{3}x + 4 \\ x &= -3y \end{aligned}$$

c. $$\begin{aligned} 3x - y &= 7 \\ y &= 3x - 2 \end{aligned}$$

d. $$\begin{aligned} x + 2y &= 1 \\ 3x + 5y &= 8 \end{aligned}$$

6-82. The Math Club is baking pies for a bake sale. The fruit-pie recipe calls for twice as many peaches as nectarines. If it takes a total of 168 pieces of fruit for all of the pies, how many nectarines are needed?

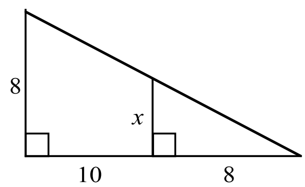
6-83. Candice is solving this system:

$$\begin{aligned} 2x - 1 &= 3y \\ 5(2x - 1) + y &= 32 \end{aligned}$$

a. She notices that each equation contains the expression $2x - 1$. Can she substitute $3y$ for $2x - 1$? Why or why not?

b. Substitute $3y$ for $2x - 1$ in the second equation to create one equation with one variable. Then solve for x and y.

6-84. Examine the diagram at right. The smaller triangle is similar to the larger triangle. Write and solve a proportion to find x.



6-85. Figure 2 of a tile pattern is shown at right. If the pattern grows linearly and if Figure 5 has 15 tiles, then find a rule for the pattern.

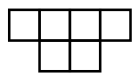
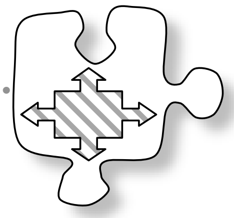


Figure 2

6-86. Given the hypothesis that line l is parallel to line m and that line m is parallel to line n, what can you conclude? Justify your conclusion.

6.3.1 What can I do now?



Pulling It All Together

This lesson contains many problems that will require you to use the algebra content you have learned so far in new ways. It will require you to use all five Ways of Thinking (justifying, making connections, applying and extending, reversing thinking, and generalizing) and will help you solidify your understanding.

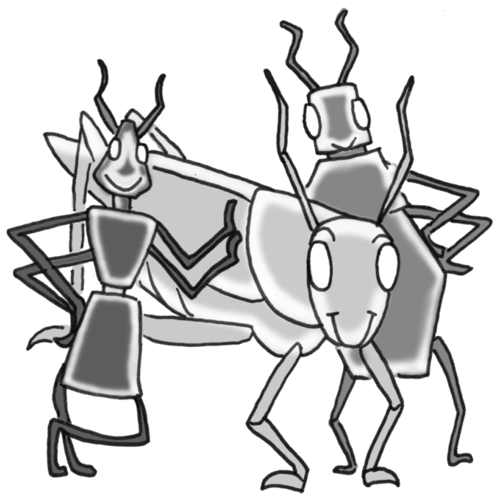
Your teacher will describe today's activity. As you solve the problems below, remember to make **connections** between all of the different subjects you have studied in Chapters 1 through 6. If you get stuck, think of what the problem reminds you of. Decide if there is a different way to approach the problem. Most importantly, discuss your ideas with your teammates.

6-87. Brianna has been collecting insects and measuring the lengths of their legs and antennae. Below is the data she has collected so far.

	Ant	**Beetle**	**Grasshopper**
Length of Antenna (x)	2 mm	6 mm	20 mm
Length of Leg (y)	4 mm	10 mm	31 mm

a. Graph the data Brianna has collected. Put the antenna length on the x-axis and leg length on the y-axis.

b. Brianna thinks that she has found an algebraic rule relating antenna length and leg length: $4y - 6x = 4$. If x represents the length of the antenna and y represents the leg length, could Brianna's rule be correct? If not, find your own algebraic rule relating antenna length and leg length.



c. If a ladybug has an antenna 1 mm long, how long does Brianna's rule say its legs will be? Use both the rule and the graph to **justify** your answer.

6-88. Barry is helping his friend understand how to solve systems of equations. He wants to give her a problem to practice. He wants to give her a problem that has two lines that intersect at the point (–3, 7). Help him by writing a system of equations that will have (–3, 7) as a solution and demonstrate how to solve it.

6-89. Examine the generic rectangle at right. Determine the missing attributes and then write the area as a product and as a sum.

	$2xy$		
-3		$-9x$	
	y		-5

6-90. One evening, Gemma saw three different phone-company ads. TeleTalk boasted a flat rate of 8¢ per minute. AmeriCall charges 30¢ per call plus 5¢ per minute. CellTime charges 60¢ per call plus only 3¢ per minute.

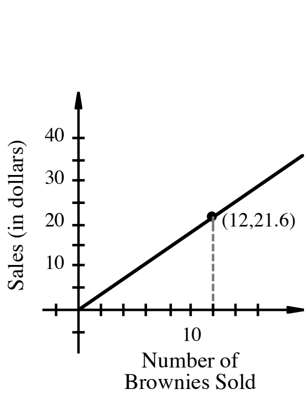
a. Gemma is planning a phone call that will take about 5 minutes. Which phone plan should she use and how much will it cost?

b. Represent each phone plan with a table and a rule. Then graph each plan on the same set of axes, where x represents time in minutes and y represents the cost of the call in cents. If possible, use different colors to represent the different phone plans.

c. How long would a call need to be to cost the same with TeleTalk and AmeriCall? What about AmeriCall and CellTime?

d. Analyze the different phone plans. How long should a call be so that AmeriCall is cheapest?

6-91. Lashayia is very famous for her delicious brownies, which she sells at football games. The graph at right shows the relationship between the number of brownies she sells and the amount of money she earns.



a. How much should she charge for 10 brownies? Be sure to demonstrate your reasoning.

b. During the last football game, Lashayia made \$34.20. How many brownies did she sell? Show your work.

6-92. How many solutions does each equation below have? How can you tell?

a. $4x - 1 + 5 = 4x + 3$

b. $6t - 3 = 3t + 6$

c. $6(2m - 3) - 3m = 2m - 18 + m$

d. $10 + 3y - 2 = 4y - y + 8$

6-93. Anthony has the rules for three lines: A, B, and C. When he solves a system with lines A and B, he gets no solution. When he solves a system with lines B and C, he gets infinite solutions. What solution will he get when he solves a system with lines A and C? **Justify** your conclusion.

6-94. Complete the Guess and Check table below and find a solution. Then write a possible word problem that would fit the table.

Stevie	**Joan**	**Julio**	**Total**	**31.50? Check**
3	5	8.50	16.50	Too low
10	19	22.50	51.50	Too high
7.50	14	17.50	39.00	Too high

6-95. Normally, the longer you work for a company, the higher your salary per hour. Hector surveyed the people at his company and placed his data in the table below.

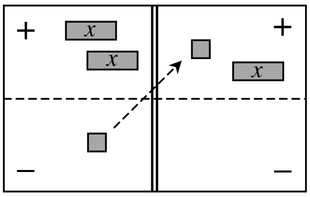
Number of Years at Company	1	3	6	7
Salary per Hour	\$7.00	\$8.50	\$10.75	\$11.50

a. Use Hector's data to estimate how much he makes, assuming he has worked at the company for 12 years.

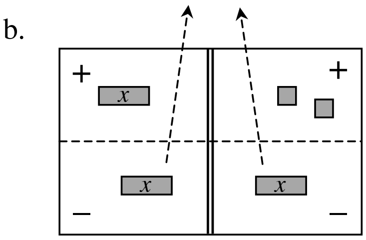
b. Hector is hiring a new employee who will work 20 hours a week. How much should the new employee earn for the first week?

6-96. Dexter loves to find shortcuts. He has proposed a few new moves to help simplify and solve equations. Examine his work below. For each, decide if his move is "legal." That is, decide if the move creates an equivalent equation. **Justify** your conclusions using the "legal" moves you already know.

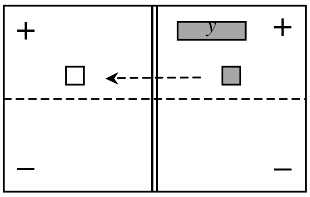
a.



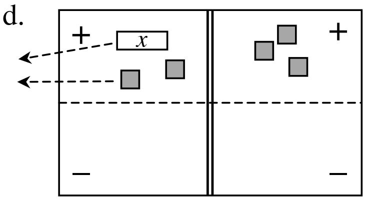
b.



c.



d.



6-97. Solve the problem below using *two different methods.*

The Math Club sold roses and tulips this year for Valentine's Day. The number of roses sold was 8 more than 4 times the number of tulips sold. Tulips were sold for \$2 each and roses for \$5 each. The club made \$414.00. How many roses were sold?

6-98. Use substitution to find where the two parabolas below intersect. Then confirm your solution by graphing both on the same set of axes.

$$y = x^2 + 5$$
$$y = x^2 + 2x + 1$$

MATH NOTES

METHODS AND MEANINGS

The Elimination Method for Solving Systems of Equations

One method of solving systems of equations is the **Elimination Method**. This method involves adding or subtracting both sides of two equations to eliminate a variable. Equations can be combined this way because balance is maintained when equal amounts are added to both sides of an equation. For example, if $a = b$ and $c = d$, then if you add a and c you will get the same result as adding b and d. Thus, $a + c = b + d$.

Consider the system of linear equations shown at right. Notice that when both sides of the equations are added together, the sum of the *x*-terms is zero and so the *x*-terms are eliminated. (Be sure to write both equations so that *x* is above *x*, *y* is above *y*, and the constants are similarly matched.)

$$\begin{array}{r} 3x + 2y = 14 \\ \underline{-3x + 5y = 14} \\ \frac{7y}{7} = \frac{28}{7} \\ y = 4 \end{array}$$

Now that you have one equation with one variable ($7y = 28$), you can solve for *y* by dividing both sides by 7. To find *x*, you can substitute the answer for *y* into one of the original equations, as shown at right. You can then test the solution for *x* and *y* by substituting both values into the other equation to verify that $-3x + 5y = 14$.

$$\begin{array}{r} 3x + 2(4) = 14 \\ 3x + 8 = 14 \\ 3x = 6 \\ x = 2 \end{array}$$

$$-3(2) + 5(4) = 14 \checkmark$$

Since $x = 2$ and $y = 4$ is a solution to both equations, it can be stated that the two lines cross at the point (2, 4).

Review & Preview

6-99. Find the point of intersection for each set of equations below using any method. Check your solutions, if possible.

a. $6x - 2y = 10$
$3x - 5 = y$

b. $6x - 2y = 5$
$3x + 2y = -2$

c. $5 - y = 3x$
$y = 2x$

d. $y = \frac{1}{4}x + 5$
$y = 2x - 9$

6-100. Consider the equation $-6x = 4 - 2y$.

a. If you graphed this equation, what shape would the graph have? How can you tell?

b. Without changing the form of the equation, find the coordinates of three points that must be on the graph of this equation. Then graph the equation on graph paper.

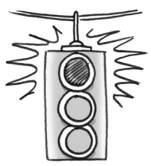
c. Solve the equation for *y*. Does your answer agree with your graph? If so, how do they agree? If not, check your work to find the error.

6-101. A tile pattern has 10 tiles in Figure 2 and increases by 2 tiles for each figure. Find a rule for this pattern and then determine how many tiles are in Figure 100.

6-102. Make a table and graph the rule $y = -x^2 + x + 2$ on graph paper. Label the *x*-intercepts.

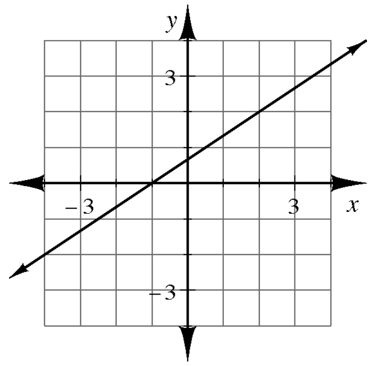
6-103. Mr. Greer solved an equation below. However, when he checked his solution, it did not make the original equation true. Find his error and then find the correct solution.

$$\begin{aligned} 4x &= 8(2x-3) \\ 4x &= 16x - 3 \\ -12x &= -3 \\ x &= \tfrac{-3}{-12} \\ x &= \tfrac{1}{4} \end{aligned}$$



6-104. Thirty coins, all dimes and nickels, are worth \$2.60. How many nickels are there?

6-105. **Multiple Choice:** Martha's equation has the graph shown at right. Which of these are solutions to Martha's equation? (Remember that more than one answer may be correct.)



a. (–4, –2)

b. (–1, 0)

c. $x = 0$ and $y = 1$

d. $x = 2$ and $y = 2$

6-106. Copy and complete the table below. Then write the corresponding rule.

IN (x)	2	10	6	7	–3	0	–10	100	x
OUT (y)	–7				18	3			

6-107. Solve the following equations for x, if possible. Check your solutions.

a. $-(2-3x)+x=9-x$

b. $\frac{6}{x+2}=\frac{3}{4}$

c. $5-2(x+6)=14$

d. $\frac{1}{2}x-4+1=-3-\frac{1}{2}x$

6-108. Using the variable x, write an equation that has no solution. Explain how you know it has no solution.

6-109. Given the hypothesis that $2x-3y=6$ and $x=0$, what can you conclude? **Justify** your conclusion.

6-110. **Multiple Choice:** Which equation below could represent a tile pattern that grows by 3 and has 9 tiles in Figure 2?

a. $3x+y=3$

b. $-3x+y=9$

c. $-3x+y=3$

d. $2x+3y=9$

Chapter 6 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

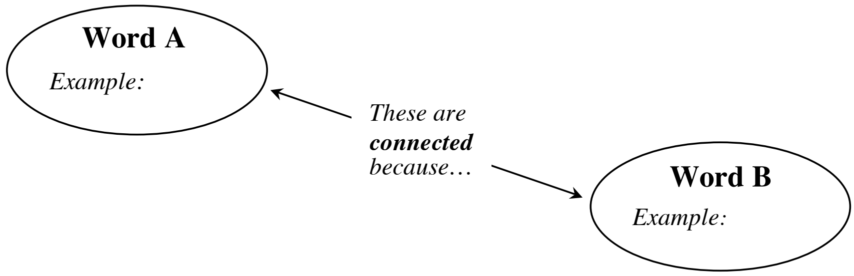
② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

coefficients	**coincide**	**Elimination Method**
Equal Values Method	equation	graph
"let" statement	linear equation	**mathematical sentence**
ordered pair	**parallel**	point of intersection
situation	solution	standard form
Substitution Method	system of equations	variable
$y = mx + b$		

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example on the following page).

② *Continued from previous page.*



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this. Your teacher may give you a "GO" page to work on. The "GO" stands for "Graphic Organizer," a tool you can use to organize your thoughts and communicate your ideas clearly.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 6-111. Solve these systems of equations using any method.

a. $y = 3x + 7$
$y = -4x + 21$

b. $3x - y = 17$
$-x + y = -7$

c. $x = 3y - 5$
$2x + 12y = -4$

d. $2x - 3y = -16$
$-4x + 2y = -4$

CL 6-112. Bob climbed down a ladder from his roof, while Rob climbed up another ladder next to him. Each ladder had 30 rungs. Their friend Jill recorded the following information about Bob and Rob:

Bob went down 2 rungs every second.

Rob went up 1 rung every second.

At some point, Bob and Rob were at the same height. Which rung were they on?

CL 6-113. Solve for x.

a. $6x - 11 = 4x + 12$

b. $2(3x - 5) = 6x - 4$

c. $(x - 3)(x + 4) = x^2 + 4$

d. $\frac{x}{25} = \frac{7}{10}$

CL 6-114. Solve the equations in parts (a) and (b) for y. Then name the growth factor and the y-intercept of each equation in part (c).

a. $-6x - 2y = 8$

b. $2x^2 + 2y = 4x + 2x^2 - 7$

c. For each of the two solved equations, find the y-intercept and growth factor. **Justify** your answers.

CL 6-115. Florida ecologists sampled Lake George to estimate the number of rainbow trout in the lake. Out of 156 fish, 18 were rainbow trout. About how many rainbow trout should they expect to find in a sample of 500 fish?

CL 6-116. As treasurer of his school's 4H club, Kenny wants to buy gifts for all 18 members. He can buy t-shirts for \$9 and sweatshirts for \$15. The club has only \$180 to spend. If Kenny wants to spend all of the club's money, how many of each type of gift can he buy?

a. Write a system of equations representing this problem.

b. Solve your system of equations and figure out how many of each type of gift Kenny should buy.

CL 6-117. Simplify each expression.

a. $3(x^2 - 7x) + 5xy - (x - 4xy) - 2x^2 + 21x$

b. $3y - (4x + 7) - y + 11 + (2x - y + 12)$

CL 6-118. Rewrite each expression below as a product and as a sum.

a. $(x+7)(2x-5)$ b. $5x(y-7)$ c. $(3x-7)(x^2-2x+11)$

CL 6-119. Each part (a) through (d) below represents a different tile pattern. For each, find the growth factor and the number of tiles in Figure 0.

a.

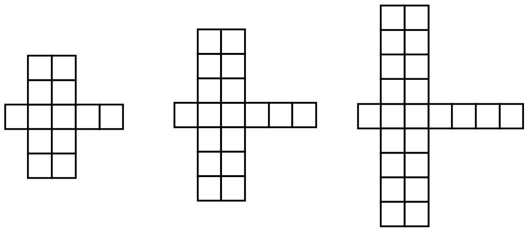
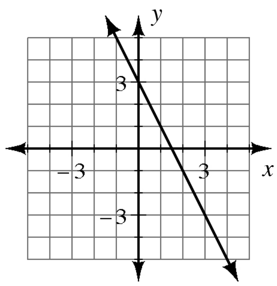


Figure 2 Figure 3 Figure 4

b.



c. $y=3x-14$

d.

x	–3	–2	–1	0	1	2	3
y	18	13	8	3	–2	–7	–12

CL 6-120. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

Review each of the Ways of Thinking that are described in the closure sections of Chapters 1 through 5. Then choose three of these Ways of Thinking that you remember using while working in this chapter. For each Way of Thinking that you choose, show and explain where you used it and how you used it. Describe why thinking in this way helped you solve a particular problem or understand something new. (For instance, explain why you wanted to **generalize** in this particular case, or why it was useful to see these particular **connections**.) Be sure to include examples to demonstrate your thinking.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 6-111.	a. $x=2$, $y=13$ b. $x=5$, $y=-2$ c. $x=-4$, $y=\frac{1}{3}$ d. $x=\frac{11}{2}$, $y=9$	Lessons 6.2.2, 6.2.3, and 6.3.1 Math Notes boxes	Problems 6-24, 6-25, 6-32, 6-34, 6-51, 6-56, 6-61, 6-62, 6-71, and 6-81
CL 6-112.	They were on the 10th rung.	Lessons 6.2.2, 6.2.3, and 6.3.1 Math Notes boxes	Problems 6-38, 6-43, 6-52, 6-58, 6-77, 6-90, and 6-97
CL 6-113.	a. $x=11.5$ b. no solution c. $x=16$ d. $x=17.5$	Lesson 5.1.3 Math Notes box, Lesson 5.1.4	Problems 6-16, 6-37, and 6-107
CL 6-114.	a. $y=-3x-4$ b. $y=2x-\frac{7}{2}$ c. (a) y-intercept: (0, –4), growth: –3 (b) y-intercept: (0, –3.5), growth: 2	Lesson 5.1.5, Lesson 5.1.5 Math Notes box	Problems 6-12 and 6-100
CL 6-115.	approximately 58 rainbow trout	Lesson 5.2.1, Lesson 5.2.1 Math Notes box	Problems 6-11, 6-28, and 6-64
CL 6-116.	a. $9x+15y=180$, $x+y=18$ b. 15 t-shirts, 3 sweatshirts	Lessons 6.2.2, 6.2.3, and 6.3.1 Math Notes boxes	Problems 6-38, 6-43, 6-52, 6-58, 6-77, 6-90, and 6-97

Problem	Solution	Need Help?	More Practice
CL 6-117.	a. $x^2 + 9xy - x$ b. $y - 2x + 16$	Lessons 2.1.5 and 5.1.3 Math Notes boxes	Problems 4-6, 3-15, and 3-75
CL 6-118.	a. $2x^2 + 9x - 35$ b. $5xy - 35x$ c. $3x^3 - 13x^2 + 47x - 77$	Lessons 5.1.3 and 5.2.3 Math Notes boxes	Problems 6-66 and 6-103
CL 6-119.	a. growth: 5, Figure 0: 3 tiles b. growth: –2, Figure 0: 3 tiles c. growth: 3, Figure 0: –14 tiles d. growth: –5, Figure 0: 3 tiles	Sections 3.1 and 4.1, Lesson 4.1.7 Math Notes box	Problems 6-10, 6-76, 6-85, 6-101, and 6-110